

Name:

Solutions

1. Find $\frac{dy}{dx}$ for the function $y = -2x^4 + x^3 - 3x^2 + 5x - 7$

$$\frac{dy}{dx} = -8x^3 + 3x^2 - 6x + 5$$

2. Find $f'(x)$ for the function $f(x) = \frac{x^3}{9} + \frac{x^2}{4} - \frac{x}{2}$

$$f'(x) = \frac{x^2}{3} + \frac{x}{2} - \frac{1}{2}$$

3. Find $\frac{dy}{dx}$ for the function $y = ax^2 + bx + c$

$$\frac{dy}{dx} = 2ax + b$$

4. Find $f'(x)$ for the function $f(x) = 6x^{-1} - 5x^{-2}$

$$f'(x) = -6x^{-2} + 10x^{-3} = -\frac{6}{x^2} + \frac{10}{x^3}$$

5. Find $\frac{dy}{dx}$ for the function $y = 2\sqrt{x} + 3\sqrt[3]{x} + 10 = 2x^{\frac{1}{2}} + 3x^{\frac{1}{3}} + 10$

$$\begin{aligned}\frac{dy}{dx} &= 2 \cdot \frac{1}{2} x^{-\frac{1}{2}} + 3 \cdot \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{2}{3}}} \\ &= x^{-\frac{1}{2}} + x^{-\frac{2}{3}}\end{aligned}$$

6. Find y' for the function $y = (3x^3 - 7x) \left(\frac{2}{x} - \frac{4}{x^2} \right)$

$$y' = (9x^2 - 7) \left(\frac{2}{x} - \frac{4}{x^2} \right) + (3x^3 - 7x) (-2x^{-2} + 8x^{-3})$$

7. Find $\frac{dy}{dx}$ for the function $y = \frac{x^3 + 1}{\sqrt{x}}$

$$\frac{dy}{dx} = \frac{\sqrt{x}(3x^2) - (x^3 + 1) \cdot \frac{1}{2} x^{-\frac{1}{2}}}{x}$$

8. Find $f'(x)$ for the function $f(x) = (x+7)^4 (x-1)^5$

$$f'(x) = 4(x+7)^3 (x-1)^5 + (x+7)^4 \cdot 5(x-1)^4$$

9. Given that functions $u(x)$ and $v(x)$ are differentiable at $x=2$ and that $u(2)=-1$, $u'(2)=4$, $v(2)=1$, and $v'(2)=-2$. Find the value of each derivative at $x=2$.

a. $\frac{d}{dx}(uv) = u'v + uv'$ At $x=2$ $\frac{d(uv)}{dx} = u'(2)v(2) + u(2)v'(2)$

$$\frac{d(uv)}{dx} = (4)(1) + (-1)(-2) = 4 + 2 = 6$$

b. $\frac{d}{dx}\left(\frac{v}{u}\right) = \frac{uv' - vu'}{u^2}$ At $x=2$ $\frac{d}{dx}\left(\frac{v}{u}\right) = \frac{u(2)v'(2) - v(2)u'(2)}{[u(2)]^2}$

$$\frac{d}{dx}\left(\frac{v}{u}\right) = \frac{(-1)(-2) - (1)(4)}{(-1)^2} = \frac{2 - 4}{1} = -2$$

10. Write an equation for the line tangent to the graph of $f(x) = -2x^3 + 10x + 5$ at $x=2$.

$$f(2) = -2(2)^3 + 10(2) + 5 = -16 + 20 + 5 = 9$$

$$f'(x) = -6x^2 + 10 \quad f'(2) = -6(2)^2 + 10 = -24 + 10 = -14$$

Tangent line Equat. $y - 9 = -14(x - 2)$

11. Write an equation for the line normal to the graph of $f(x) = x^2(x+3)^3$ at $x=-1$

$$f(-1) = (-1)^2(-1+3)^3 = (2)^3 = 8$$

$$f'(x) = 2x(x+3)^3 + x^2(3(x+3)^2)$$

$$f'(-1) = 2(-1)(-1+3)^3 + (-1)^2(3(-1+3)^2) = -2(8) + 3(4) = -4$$

Normal
Line
Equation

$$y - 8 = \frac{1}{4}(x + 1)$$

